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Partially Latent Interaction in Elementary Particle Formation

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Abstract

In order to surmount unending difficulties currently encountered by strong interaction theory, drastic modifications appear to be needed. A possible modification is conjectured by introducing the concept of quasidynamical strong interaction at very short distances. The feasibility of tightly binding quasidynamical $\bar{K}$–N interaction to deliver hyperons is considered. The quasidynamical interaction, if proved correct, may provide an essential interaction mechanism for the formation of strongly interacting particle states.

1. *Introduction*

Quantum theory has not yet been proved to be valid universally. Current theory yields agreement with experiment to a limited degree of approximation only for a restricted domain of physical phenomena. Difficulties encountered with current theory are especially pronounced in the domain of strong interactions at very short distances.

Possible improvement of current strong interaction theory, now considered as acutely desirable, can be accomplished along one of the following lines:

(1) Further modification without straying from the currently accepted basic quantum theoretical principles.
(2) Radical reformulation, or even abandonment, of some of the currently accepted basic quantum mechanical principles.

As to the first approach, various elaborate ideas have already been extensively explored, but to little avail. The need for a fundamental change in current theory along the line of the second approach, has been increasingly and convincingly advocated. However, few have so far advanced concrete new ideas in this direction.

One of the basic assumptions in current theory is that the physical states are exhaustively describable in terms of 'observables', represented by Hermitian operators. However, can the human mind unequivocally claim such 'observational' accesses to every essential aspect of the workings of nature? Such an unqualified assertion may not be entirely warranted at this time, especially in view of the unending difficulties encountered with current strong interaction theory. History indicates many breakdowns of basic scientific hypotheses, although often believed to be wholly true, when confronted with newly developed situations. Therefore, a cross-examination of the requirement of 'observables' that has been universally accepted without truly valid proof is now in order, especially as the workings of nature are being probed more deeply into the still unknown domain.

The 'observables' of particle states in current theory are related externally to a measuring apparatus, and not necessarily to the wholly inclusive inner characteristics of the particle states themselves.† It is entirely conceivable that physical principles in the deeper domain of nature differ markedly from those currently accepted as universally valid, and they may reveal further inner properties that cease to be undividedly 'observable'. This possibility would extend the realms of the quantum theory to the inclusion of non-Hermitian operators.

As a first step exploration in this direction, an anti-Hermitian operator Ω may be contained in a state Hamiltonian‡ in the form of Ω^2. The interaction may yield meaningful physical states, if the consequences of such description are not only generally compatible with the rest of the basic quantum principles, but also if the inclusion of the non-Hermitian operator Ω helps eradicate the difficulties encountered in current theory. States involved in such interaction will be designated as 'quasidynamical'.

2. *Quasidynamical $\bar{K}$–N Interaction*

Such a radical modification of current theory should be geared to a search for its clue in actual physical situations. An appropriate area in which to begin the search, lies in the conceivably simple interaction mechanism which, nevertheless, could signal a breakdown of current theory.

Consider, for instance, the $\bar{K}$–N interaction and forthrightly assume that the $\bar{K}$–N interaction creates the Λ or Σ hyperon state. One of the favorable features of this assumption is that both $\bar{K}$ and N have been experimentally observed and their properties are reasonably well understood. Furthermore, the tightly bound $\bar{K}$–N system is in many respects comparable with the (Λ, Σ) hyperon states.

The most provocative feature that led to the consideration of this particular interaction, however, is that the $\bar{K}$–N bound (Λ, Σ) hyperon states also yield, as will be shown, consequences that are difficult to reconcile to a simple interpretation within current theory.§ Indication of difficulties such

† For an illuminating discussion, see Bohm, D. (1957). *Causality and Chance in Modern Physics*. Routledge and Kegan-Paul, London.

‡ The Hamiltonian thus remains Hermitian.

§ This is based on the premise that the fundamental laws of nature are simple in essence, and thus a correct theory necessarily enables a qualitatively correct simple interpretation of the consequences. Although such a premise appears to be quite stringent at this time, the quasidynamical interaction conjectured for the $\bar{K}$–N bound states in this paper indicates that such a simple interpretation can be possible.

as this may be analyzed for a possible clue for a radical modification of current theory. Note here that the $\bar{K}$–N bound (Λ, Σ) hyperon state interaction is strong† and that its effective interaction would take place at radius $\langle r_{\bar{K}N}\rangle \approx 1/\mu_{\bar{K}N}$, which is much smaller than $\langle r_{\pi N}\rangle \approx 1/\mu_{\pi N}$. Here, the $\bar{K}$–N and π–N reduced masses are denoted, respectively, by $\mu_{\bar{K}N}$ and $\mu_{\pi N}$. Precisely such strong interaction at a very short distance is the domain where, as indicated above, the difficulties in current theory are pronounced. Therefore, some clear-cut breakdown of current theory could conceivably occur in these $\bar{K}$–N bound hyperon states.

Now, assume that the $\bar{K}$–N bound state Hamiltonian‡ is approximated by

$$H = p_r^{\,2} + \frac{\bar{L}^2}{r^2} + V_\mu(r) + V_c(r) \tag{2.1}$$

Here, $V_\mu(r)$ represents the Yukawa potential energy $-q_{\bar{K}N}^2 \exp(-\mu r)/r$, with $\mu = \mu_{\bar{K}N}$, and $V_c(r)$ represents a strongly repulsive core potential energy that is a physical requisite between the interacting particles at a very short distance.

The linear momentum component p_r and the angular momentum $\bar{L}$ enter the Hamiltonian in the form of $p_r^{\,2}$ and $\bar{L}^2$. Thus the Hamiltonian remains Hermitian even if p_r and/or $\bar{L}$ become anti-Hermitian. However, by examining the Schrödinger equation [equation (3.1)], it is readily seen that p_r must be Hermitian to produce a bound state, while $\bar{L}$ needs not be. When $\bar{L}$ is anti-Hermitian, by introducing a Hermitian operator $\bar{I}$ by $\bar{L} = i\bar{I}$, the structure of rotation can be equivalently formulated (Hamermesh, 1962), as it is in the Hermitian $\bar{L}$ case.

Note that the quasidynamical interaction to be conjectured here with an introduction of the anti-Hermitian $\bar{L}$ may have been imbedded in current theory itself. The uncertainty principle indicates that, when the interaction distance becomes smaller, the uncertainty in the momentum gets increasingly larger. The large uncertainty in the momentum may point toward a larger imaginary part§ of the momentum. This may in turn yield, as indicated above for possible formation of the bound states, large I which is inversely related to the angular width in terms of the uncertainty principle. As the interaction distance approaches the $\bar{K}$–N interaction range, the I becomes large enough that the quasidynamical quantization can presumably become accessible. More of this aspect will be discussed in Section 4, where it will be shown that the introduction of the anti-Hermitian $\bar{L}$ here‖ not only yields consistent consequences, but also that

† The approximate relation $q_{\bar{K}N}\,\mu_{\bar{K}N} \approx q_{\pi N}\,\mu_{\pi N}$, with $q_{\bar{K}N}$ and $q_{\pi N}$ respectively representing the $\bar{K}$–N and π–N interaction constants, indicates that the $\bar{K}$–N interaction is as strong as the π–N interaction.

‡ Insight into the relativistic consequences is hoped to be gained from this nonrelativistic consideration.

§ The occurrence of a complex $\bar{p}$ is by no means new; in various physical situations where the spatial dispersion arises, $\bar{p}$ is always complex.

‖ Consequences of this interaction would be partially latent.

it may eradicate some of the difficulties with current theory that would otherwise be encountered.

The requirement of 'observables' in current theory would then be asymptotically† valid as the distance of the interaction becomes large. This view is considered in parallel with the validity of classical physics in the limit of $\hbar \to 0$. If proved correct, therefore, the quasidynamical interaction would be no more peculiar than, for example, the recognition of the discontinuity of physical quantities in the unit of $\hbar$ in the quantum levels, in spite of the long-held belief that the variations in the physical quantities are absolutely continuous.

With the anti-Hermitian $\bar{L}$, the angular term in equation (2.1) produces an attractive force‡ that is sufficiently strong, but being appropriately balanced by the repulsive core $V_c(r)$ at a very short distance, to tightly bind the $\bar{K}$–N system to deliver the (Λ, Σ) hyperons. It is remarkable that, besides the familiar potential energies $V_\mu(r)$ and $V_c(r)$ already given in equation (2.1), there is no further necessity of supplementing other artificially strong attractive potential energies§ to give the large negative energy required for the $(\bar{K}\text{–}N) \to (\Lambda, \Sigma)$ binding. Free to mastermind the workings of nature, would not the Creator again choose the essentially simple course, and thus wisely command action on $\bar{L}$ here, rather than on some more complex alternative? The success that has been enjoyed by current theory in the limited domain by confining itself to the Hermitian operators, constitutes an insufficient logical basis for shunning entirely the possibility of $\bar{L}$ becoming anti-Hermitian, especially when a newly developed situation seems to make that particularly fitting.

3. *Schrödinger Equation*

Following the general procedure of coordinate separation for the wave function,

$$\Psi = \frac{U_I(r)}{r} \exp(-W^{1/2} r)\, Y_{IM}(\theta, \phi)$$

where $W = -E$, the Schrödinger wave equation for the quasidynamical $\bar{K}$–N interaction characterized by the Hamiltonian in equation (2.1) with $\bar{L} = i\bar{I}$, is reduced to

$$\frac{d^2 U_I(r)}{dr^2} - 2W^{1/2} \frac{dU_I(r)}{dr} + \left[\frac{I(I+1)}{r^2} - V_\mu(r) - V_c(r)\right] U_I(r) = 0 \quad (3.1)$$

† This premise may be valid, depending on the nature of interaction involved.

‡ This attractive force can be conceptionally understood by noting that, when $\bar{L}$ is anti-Hermitian, the energy of the state is accordingly reduced. This boosts the effective attractive force to give a tighter binding. See also equation (3.1).

§ No additional hypothetical particle in the $\bar{K}$–N interaction is implied here.

and

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]Y_{IM}(\theta,\phi)=-I(I+1)\,Y_{IM}(\theta,\phi)$$

where units $\hbar = 1$ and $2\mu = 1$ are used.

By examining the radial wave equation (3.1) as $r \to 0$ with an appropriate repulsive core $V_c(r)$ at a very short distance, it can be readily seen that the radial wave equation (3.1) can yield proper solutions for the negative real energy states. For an appropriate, but otherwise arbitrary $V_c(r)$, however, the radial wave equation (3.1) is not, in general, readily accessible to simple solutions.

Therefore, strictly for the purpose of demonstrating the possibility of obtaining solutions, take

$$V_c(r) = \mathscr{I}\left[\frac{A(A+1)}{r^2}\exp(-M_c r)\right] \tag{3.2}$$

where $A > I$ and $M_c \gg \mu$. With properly large A and M_c, the repulsive core potential energy $V_c(r)$ would impulsively affect the radial wave equation (3.1) at a very short distance. The nature of $V_c(r)$ under consideration may thus be crudely approximated (for the sake of simplicity) as the following. The repulsive core term $V_c(r)$ simply affects, in the form of $A(A+1)/r^2$, the determination of the lowest-order term (which is dominantly important at a very short distance) in the series solution of the radial wave equation (3.1); otherwise it has an insignificant effect. This simple approximation for $V_c(r)$ is explicitly indicated by an impulse operator $\mathscr{I}$ in equation (3.2). Since the strongly attractive angular term yields a tight binding, giving a large W, the effect of the Yukawa potential energy in the radial wave equation (3.1) will also be approximated (again for the sake of simplicity) in the limit of $\mu \to 0$.

With the approximations given above, a series solution

$$U_I(r) = \sum_{k=0} C_k r^{s+k}$$

for the radial wave equation (3.1) can be obtained, yielding the approximate energy eigenvalues,

$$E_{I,n} \approx -\frac{q_{KN}^4}{[\{1+4A(A+1)-4I(I+1)\}^{1/2}+2n-1]^2} \tag{3.3}$$

for $n = 1, 2, 3, \ldots$, with

$$s \approx \tfrac{1}{2}[1+\{1+4A(A+1)-4I(I+1)\}^{1/2}] > 1$$

Because of the nature of approximation employed, the result given above will probably give qualitatively correct descriptions only for the lowest (nonzero) I state.

4. *Physical Implication*

Briefly consider the possible physical implication of the quasidynamical $\bar{K}$–N interaction described above. For this purpose, note that the domain of the coordinate angle θ is not only finite,† ranging only over $0 \leqslant \theta \leqslant \pi$, but also that the usual quantum description of the angular distribution function by $|Y_{lm}(\theta,\phi)|^2$ for the real angular momentum (l,m) state, further confines it within as narrow angular stripes as $\Delta\theta \approx 1/l$. On the other hand, the uncertainty principle stipulates that the quasidynamically interacting particle would be confined to the angular stripes for $\Delta\theta \approx 1/I$, giving $\Delta\theta \approx 1$ for the $I = 1$ state. Hence, the quasidynamically interacting state, being extra-tightly bound in angular stripes compatible with the uncertainty principle, may be led to be meaningfully described by $Y_{IM}(\theta,\phi)$.

The $I = 1$ state interaction, characterized by $Y_{I=1,M}(\theta,\phi)$, introduces an additional negative parity. Thus, the quasidynamical $\bar{K}$–N interaction in the $I = 1$ state would have, as a whole, a positive parity. However, the quasidynamical quantum number I would not correlate‡ with the usual real angular momentum l. Therefore, the quasidynamically interacting $\bar{K}$–N system in the $I = 1$ state would explicitly behave as the spin-$\frac{1}{2}(l = 0)$ state, yielding correct quantum numbers for the (Λ,Σ) hyperon states as observed.

This is clearly in contrast to the consequences of ($l = 0$, parity $= -$) and ($l = 1$, parity $= +$) that would have resulted for the bound $\bar{K}$–N state in terms of current theory. The $\bar{K}$–N bound ($l = 0$, parity $= -$) state is contradictory to the observed $(J = \frac{1}{2}, \text{parity} = +)_{\Lambda,\Sigma}$ hyperon states. As to the $\bar{K}$–N bound ($l = 1$, parity $= +$) state, it appears to be unnecessarily obscure, to say the least, that the lowest-energy $\bar{K}$–N bound state is in the $l = 1$ state, rather than in the $l = 0$ state. This is because the $\bar{K}$–N binding interaction in the $l = 1$ state requires, due to its interaction at a very short effective distance, a pointlessly large additional attractive force (whose origin is entirely obscure) to counter the p-wave centrifugal force which is very large and strongly dominant§ over the attractive Yukawa force. Such a consequence is in marked contrast to the particularly simple quasi-

† Note the marked contrast with the particle lifetime $\tau \approx 1/\Gamma$ that spans over $0 < \tau < \infty$. Here, Γ, representing the uncertainty in energy, is required to vanish for a real negative energy state.

‡ Note that the particles involved are not executing rotation in the usual sense. The quasidynamical angular confinement function $Y_{IM}(\theta,\phi)$, based on the latent aspect of the interaction, would be much less conspicuous in exhibiting dynamical properties than the usual angular eigenfunction $Y_{lm}(\theta,\phi)$. The quasidynamical states may occur predominantly in the ($I = 1$, $M = 0$) state.

§ Note that the ratio (absolute value) between the effective Yukawa potential term and the effective p-wave centrifugal force term is 5·5 for the π–N interaction, while the corresponding ratio for the $\bar{K}$–N interaction is 0·7, which is smaller than 1. Therefore, while the Yukawa potential alone is sufficiently strong to bind the π–N interaction in a p-wave, the Yukawa potential is far too weak to establish the $\bar{K}$–N bound hyperon (Λ,Σ) states in a p-wave.

dynamical $\bar{K}$–N binding interaction presented in this paper for the formation of the (Λ, Σ) hyperon states.

The individuality of each of the quasidynamically bound particles is expected to be somewhat more extensively blurred as compared with that of particles participating in the usual binding mechanism. Therefore, the hyperons formed by the quasidynamical $\bar{K}$–N interaction could be expected to behave, as observed despite their comparatively short lifetime, as nearly elementary as N itself. The quasidynamical interaction would in general exhibit properties that deviate markedly from those expected in the usual interaction.

The simplest conceivable interaction mechanism for providing the main mass difference between the Λ and Σ states now appears to be the isotopic spin vector coupling.† If the Ξ state is similarly attributed to the quasi-dynamical $\bar{K}$–N–$\bar{K}$ interaction, and the effect of isotopic spin vector coupling is taken into account, the mass centers of N, (Λ, Σ) and Ξ states are separated approximately by $\Delta M \approx 235$ Mev. Note here that the energy eigenvalue from equation (3.3) for the $(I = 1, n = 1)$ state is $E_{1,1} \approx -260$ Mev, when evaluated with $A \approx 1{\cdot}30$ and $q^2_{\bar{K}N} \approx 2{\cdot}0$. This eigenvalue $E_{1,1}$ yields the same separation for the mass centers of the baryon octet states, i.e., $\Delta M \approx M_K + E_{1,1} \approx 235$ Mev. This consideration can be extended to the baryon decuplet and other multiplets, where more complicated interactions are involved.‡

5. *Conclusion*

It is logically conceivable that the workings of nature have other inner properties than those that can be described in terms of the so-called 'observables' of current theory. The quasidynamical interaction considered in this paper clearly demonstrates one possible aspect of the latent inner properties, whose influence may become increasingly apparent as the workings of nature in the strong interaction are probed more and more deeply into the short-distance domain.

The containment of an anti-Hermitian $\bar{L}$ in the quasidynamical interaction is formulated without explicitly involving imaginary quantities. A quite consistent interaction mechanism is evolved from this quasidynamical interaction for the $(\bar{K}$–$N) \rightarrow (\Lambda, \Sigma)$ binding. It is possible that, with this remarkable interaction mechanism available, the strongly interacting particle states can be forthrightly constructed from those low-mass particles

† The isotopic spin vector coupling constant for the $\bar{K}$–N interaction turns out to be approximately 39 Mev, which is much smaller than $\Delta M \approx 235$ Mev. It is easy to see that the mass formula for the octet, when deduced from the isotopic spin vector coupling, is identical to the first-order $SU(3)$ mass formula. For further discussion, see Suh, K. S. (1960). *Bulletin. American Physical Society*, **5**, 255 and Suh, K. S. (1968). *Inf. Zur Kernforschung und Kerntechnik*, **11-8302**, 137.

‡ These interactions include, besides those of $\bar{K}$ and N considered in this paper, various nonquasidynamical interaction of an increasing number of particle elements. See footnote above.

already observed, rather than from those that are unknown or unobserved. Consequently, the concept of the quasidynamical interaction constitutes a strikingly appealing prospect for providing an essential interaction mechanism for the formation of strongly interacting particle states at short distances.

Truly valid evaluation of the new concept introduced in this paper, will take much painstaking exploration. Therefore, without pretending to present a clear-cut explanation, this paper simply has raised a logically conceivable conjecture that, if proved correct, should have a profound effect on the theory of elementary particles. It is a small but potentially essential step in view of the unending difficulties encountered in current theory.

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Reference

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